

# CHAPTER 123

## GROUND-BORNE VIBRATION TRANSMISSION FROM ROAD AND RAIL SYSTEMS: PREDICTION AND CONTROL

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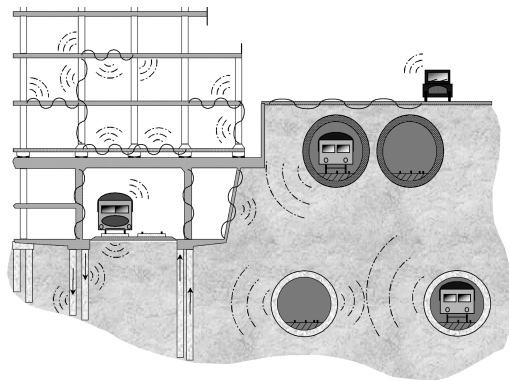
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### 1 INTRODUCTION

Vibration is generated by road and rail traffic over a wide range of frequencies and amplitudes. Large-amplitude vibration can cause damage to track components, cracking of roadways, settlement of foundations, destabilization of embankments, and damage to nearby structures. But the focus of this chapter is on disturbance due to small-amplitude vibration. Vibration is perceptible at amplitudes several orders of magnitude below those generally required to cause damage to buildings. Regulations prescribe acceptable levels of vibration in the vicinity of new roads and railways and, in particular, near the tunnels of underground railway systems in cities. It is desirable to model accurately the generation and propagation of vibration because permission to build may only be granted if vibration can be shown to be within acceptable limits. There may also be restrictions applied if traffic patterns change, for instance, if nighttime freight trains are to be introduced on a line that normally carries only daytime passenger trains or if heavy bus traffic is to be increased on a residential road. This chapter addresses the issues confronting practicing engineers when asked to evaluate or mitigate ground vibration generated by road and rail traffic.

### 2 IMPACT OF GROUND-BORNE VIBRATION

Vibration generated by road vehicles and by trains can have significant environmental impact on nearby buildings. The general complexity of the problem is illustrated in Fig. 1. Inhabitants perceive vibration either directly as motion in floors and walls or indirectly as reradiated noise. A third and very significant source of disturbance is due to movement of household objects, especially mirrors, or by the rattling of window panes and glassware. In all these cases the problem of ground-borne vibration is important at frequencies typically up to 200 to 250 Hz.<sup>1</sup> Vibration at higher frequencies is generally attenuated rapidly with distance along the transmission path through the ground. Vibration can travel long distances from its



**Figure 1** Sources of ground vibration and their transmission paths. (Courtesy of J. P. Talbot, Atkins Consultants, UK.)

source. For a ground with soft clay or silt, ground-borne vibration may produce annoyance to people in buildings more than 200 m away from tracks.<sup>2</sup>

Many studies and standards address the effect of vibration on buildings, their occupants, and on equipment. The British standard BS 6472:1992<sup>3</sup> provides general guidance on human exposure to vibration in the frequency range 1 to 80 Hz. It provides curves for equal annoyance for humans plotted as root-mean-square acceleration versus frequency. The vibration dose value (VDV) is used as a vibration measure that takes into account the time history of the vibration (whether continuous or intermittent) for the day or the nighttime. The American National Standards Institute (ANSI) annoyance criteria due to vibration<sup>4</sup> are described by the maximum weighted acceleration level and are given in Table 1.

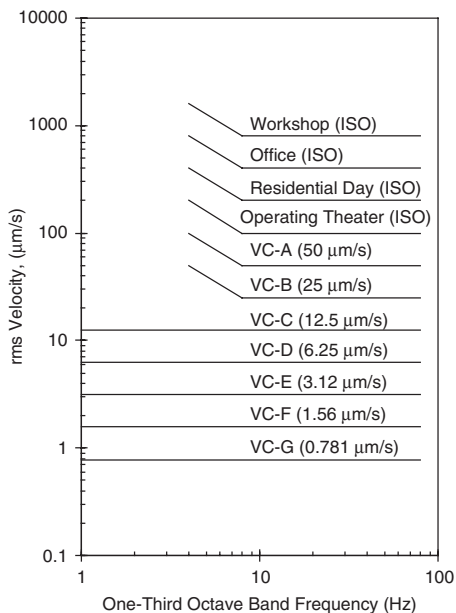
Experimental work by Duarte and Filho<sup>5</sup> shows that the sensitivity of people to a sinusoidal vibration decreases with frequency up to around 40 to 50 Hz. This is because the human head resonates around 20 to

**Table 1 ANSI Criteria for Annoyance due to Ground-Borne Vibration**

| Building Use Category       | Maximum Weighted Acceleration Level (dB re $10^{-6}$ g) |
|-----------------------------|---|
| Hospital and critical areas | 51  |
| Residential (nighttime)     | 54  |
| Residential (daytime)       | 57  |
| Office                      | 63  |
| Factory                     | 69  |

40 Hz, and hence it makes the person sensitive to even low amplitudes of vibration. Sensitivity increases again in the range between 50 and 100 Hz where the chest wall and ocular globe resonate. The work shows also that women are more sensitive to vibration than men.

Gordon<sup>6</sup> develops generic vibration criteria (VC curves) for vibration-sensitive equipment. These curves are plotted in a similar way to the International Organization for Standardization (ISO) 2631-2:2003 guidelines for the effects of vibration on people in buildings.<sup>7</sup> Vibration is expressed in terms of its root-mean-square velocity and is plotted as a one-third octave band. For a site to comply with a particular equipment category, the measured one-third octave band velocity must lie below the appropriate curve for the given category in Fig. 2.



**Figure 2** Generic vibration criterion (VC) curves for vibration-sensitive equipment showing also the ISO guidelines for people in buildings. [Courtesy of the Institute of Environmental Sciences and Technology (IEST), Rolling Meadows, Illinois; www.iest.org.]

Due to the increasing public sensitivity to noise and vibration,<sup>8</sup> railway companies must develop affordable vibration countermeasures and predictive tools to permit estimation of vibration levels in the early stages of planning for new railways. With increasing complaints about ground-borne vibration, many surveys are conducted to assess the vibration effect on buildings and their occupants and to help establish standards. In Nagoya, for example, the city authority conducts surveys about every 5 years to investigate the state of noise and vibration environment.<sup>9</sup>

A noise and vibration study by Fields<sup>10</sup> based on interviews with people who live near railway tracks finds a generally high level of dissatisfaction within 25 m of the track. People interviewed in the range 25 to 150 m from the track report rapidly diminishing levels of complaint, and for distances beyond 150 m there is a uniform and low level of complaint. The study shows that many factors affect perception such as the duration of vibration, the time of day, background vibration levels, and various psychological factors such as whether the railway is visible.

It is generally believed that damage to buildings is associated with vibration, but this is unlikely as vibration generated by road traffic or railway trains rarely approaches the threshold of architectural damage.<sup>11,12</sup> There are exceptions, and certain historic masonry structures are thought to have been damaged by traffic-induced vibration. Fear of damage to historic structures in Stockholm's medieval quarter Gamla Stan has been one of the obstacles to the building of a new track.<sup>13</sup> The fear is arguably justified in cases where vibration has the potential to cause compaction and differential settlement of foundations of buildings close to roads or railways. This has been put forward as an explanation for the road-ward inclination of cathedrals and other ancient buildings.<sup>14</sup>

Turunen-Rise et al.<sup>15</sup> and Klæboe et al.<sup>16</sup> present a new Norwegian standard NS8176 for vibration in buildings from road and rail traffic. The standard introduces a single quantity to describe the vibration in buildings. This quantity is the statistical value of maximum velocity or acceleration,  $v_{W,95}$  and  $a_{W,95}$ , respectively. These are calculated by recording the vibration for at least 15 passing trains or vehicles. For each record the third octave band frequency spectrum is calculated and weighted with weight values proportional to the human response at each band. Assuming a lognormal distribution of the root-mean-square values,  $v_{W,95}$  and  $a_{W,95}$  are calculated from the velocity and acceleration with a nonexceeding probability of 95%. In this standard vibration should be measured in a position and direction in the building that gives the highest vibration. A survey is conducted by questioning people who live in buildings where  $v_{W,95}$  and  $a_{W,95}$  were calculated using a prediction model developed by Madshus et al.<sup>2</sup> (see Section 5). According to the survey, it is found that there is no significant difference in reactions to vibration from different sources. Buildings are classified into four categories (A to D) according to the study, describing the building state in terms of vibration. In order to

provide a common answer format for sociovibrational studies and a better data exchange from future surveys to assess human perception of vibration in buildings. Klaboe et al.<sup>17</sup> present a methodology to standardize carrying out a survey.

Research on ground-borne vibration, especially from underground railways, has recently gained prominence on account of the need to establish new underground tunnels in cities. There is also pressure to put high-speed intercity lines underground near residential areas. A particular feature of underground trains is the widespread notion that once underground the problem of noise goes away. But pure vibration in the absence of noise can be unnerving and more disturbing than vibration from visible and audible surface trains. Also of concern is that underground tunnels may pass near building foundations causing significant structural vibration. To address these issues a program of research entitled CONVURT (control of vibration from underground rail traffic) was established under the fifth framework of the European Union (EU) program for research, technological developments, and demonstration. The project had four main objectives: The first was to create validated innovative and quantitative modeling tools to enable accurate prediction of ground-borne vibration transmission into buildings.<sup>18,19</sup> The second was to develop and evaluate new and cost-effective track and tunnel components to reduce ground-borne vibration and especially to develop devices capable of being retrofitted to existing track.<sup>20</sup> The third was to provide scientific input to allow the preparation of international standards. Finally, CONVURT has established guidelines of good practice for underground railway operation in order to maintain minimum vibration for the lifetime of operation.<sup>21</sup>

### 3 EXCITATION MECHANISMS

Vibration from road and rail vehicles derives from the inertia forces generated as vehicles pass over road and track irregularities. The nature of irregularities differs significantly between road and rail vehicles. For road vehicles much attention is given to potholes and speed cushions. A typical study is carried out by Watts<sup>22</sup> to assess the vibration levels generated by vehicle crossing speed control cushions and road humps finds that these speed control measures can produce perceptible levels of vibration, but it is very unlikely to cause even minor damage to buildings. In general the resilience offered by pneumatic tires helps to reduce the generation of high dynamic forces and so reduce the level of traffic-induced vibration in the vicinity of roads. By contrast the dynamic forces generated by heavy steel wheels rolling on steel rails are high, and the rest of this section will address excitation mechanisms for rail vehicles.

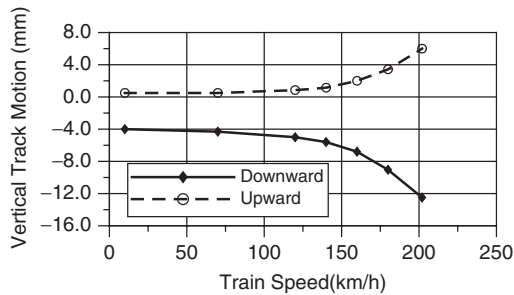
Significant vibration in buildings near railway tracks and subway tunnels is attributed to moving trains. There are five main mechanisms for the generation of vibration. The first is a *quasi-static* mechanism, and it is significantly close to a track where the passage of individual axles can be distinguished. The passage of a vehicle can be represented by the passage of a

number of discrete concentrated static forces applied to the rails. As each force passes by, the observer experiences an oscillatory motion even though the applied force is constant. This may be visualized by imagining a child riding a bicycle over a large mattress; the vertical applied force is a constant, yet oscillatory motion is induced in objects resting on the mattress. The quasi-static effect contributes significantly to the low-frequency response in the range 0 to 20 Hz.<sup>1,23</sup>

A second mechanism, known as *parametric excitation*, derives from the periodic support of rail by sleepers. The mechanism is similar to the quasi-static effect described above because vibration is generated when a constant wheel load moves along the rail at constant speed. The difference is that when it encounters variable rail support stiffness the resulting vertical motion of the wheel and axle cause dynamic forces to be applied to the rail. This is analogous to the same bicycle being influenced by individual springs in the mattress and the vertical applied force has an oscillatory component. Sleepers are uniformly spaced and excitation occurs at a frequency equal to the train velocity divided by the sleeper spacing (typically 25 Hz for low-speed trains and up to 150 Hz for high-speed trains). An investigation of this effect by Heckl et al.<sup>24</sup> gives results for the measured acceleration spectra near a railway track. It shows that peaks appear at some distinctive frequencies such as the sleeper-pass frequency, and also that the response can be large if the wheel track resonance coincides with one of the sleeper-pass harmonics.

A third mechanism occurs when wheels apply impulsive loading to the rail due to height differences at rail joints and crossings. This mechanism is becoming less important with the increasingly widespread use of continuously welded track, but the growth of surface unevenness is often more rapid in the vicinity of welds. Impulsive loading also occurs when the wheels have "flats" caused by skidding with locked brakes.

The fourth and generally most important source of vibration is that generated by rail and wheel unevenness or roughness. The unloaded rail profile is not smooth, and this is true even when the rail is new and has been recently installed. The reason is that the track and trackbed are never perfectly straight and flat. With time, ballast degrades and wheels and rails deteriorate so that rail roughness increases. Rail roughness contributes to vibration generation over a wide range of frequencies. Typical rail roughness has higher amplitudes for long wavelengths. An exception to this rule is that a major source of rail roughness is *corrugation* at wavelengths typically around 25 to 50 mm,<sup>25</sup> but for typical train speeds these short wavelengths generate vibration at frequencies well above 200 Hz. At low speeds (speed-restricted track and at the approaches to stations) dynamic forces due to corrugation are generally small in amplitude unless the track is badly corrugated. It is not generally necessary to consider corrugation when evaluating ground vibration from railways. Wheel roughness generally contributes to perceived roughness uniformly across the frequency range of interest to ground



**Figure 3** Average measured downward and upward vertical displacement of a track with different train velocities. (Reproduced from Ref. 28, by permission.)

vibration.<sup>26</sup> There is a good summary of the various factors contributing to rail roughness by Hunt.<sup>27</sup>

The fifth and final mechanism by which very large amplitudes of vibration can be generated occurs when train speeds approach or exceed either the speed of Rayleigh surface waves in the ground or the minimal phase velocity of bending waves in the track; see Fig. 3. In low-speed urban rail networks no attention need be given to this mechanism because the speed of trains is low compared with all critical wave propagation speeds. In recent years some attention has been given to this mechanism due to the increasing international trend toward higher speed trains; see, for example, Refs. 29 and 30. For the case of very soft soil, critical speed (often the Rayleigh wave speed) can be easily exceeded by modern trains. If the train velocity exceeds the Rayleigh wave speed, a ground vibration boom occurs. In a location near Ledsgard in Sweden the Rayleigh wave speed is around 45 m/s. An increase in train speed from about 38 to 50 m/s led to an increase of about 10 times in the generated ground vibration. If train speed increases further to the point where it reaches the minimal track phase velocity, larger deflections can occur with potential for train derailment. Vibration boom also occurs for underground trains if the speed reaches the velocity of shear waves in the ground.

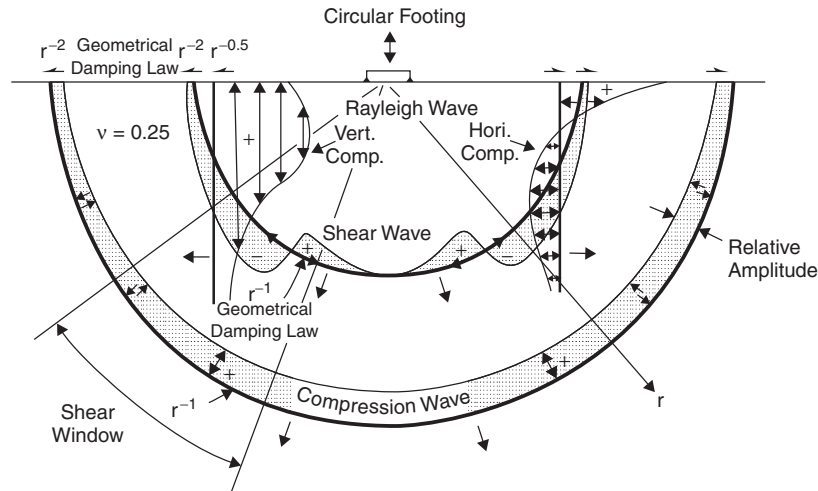
There are other special sources of vibration excitation in addition to the five mentioned above. For instance, vibration is generated when a train approaches a bridge due to the change of trackbed geometry and stiffness.<sup>31</sup> However, stiffness variation can be classified as a parametric excitation. There are also some avoidable sources of vibration, for instance, those due to static and dynamic out-of-balance of wheels, shafts, gearboxes, motors, and couplings aboard the train. Also there are disturbances that arise from tracking instability, cornering, flange contact, and gauge variation. These sources are generally of concern in older and poorly maintained track and are beyond the scope of this chapter.

#### 4 MODES OF PROPAGATION

Dynamic forces generated at the road-wheel or rail-wheel interface propagate as vibration through

the ground and into buildings, causing annoyance to people. Vibration waves in the ground are superficially like sound waves in air except that in solids many different kinds of waves exist depending on the nature of the medium and its boundaries. Even in an isotropic elastic full space (the simplest of solid media being an elastic solid infinite in all directions), two kinds of waves can propagate, and these are called the body waves, moving spherically away from a localized point of excitation. The first is a dilatational wave, or a *pressure* wave (known as the P-wave). This is a wave similar to compressive waves in air where the particles within the medium oscillate parallel to the direction of propagation of the wavefront. The second is the equivoluminal wave, or the *shear* wave (known as the S-wave). This is a transverse wave where particle motion is perpendicular to the direction of travel of the wavefront. There is no equivalent to the S-wave in air because air possesses no stiffness in shear. The situation becomes more complicated when one considers that an elastic full space has no free surface and is therefore unsuited to modeling wave propagation in the ground. It is more common to use models based on an elastic half-space (see Fig. 4) where a third kind of wave appears, confined to the free surface. This is the *Rayleigh* wave, also known as the R-wave, and the motion of particles is elliptical in a plane perpendicular both to the free surface and to the wavefront. Rayleigh waves are superficially similar to surface waves in water, but, whereas water waves are controlled by the action of gravity or surface tension, the Rayleigh wave is controlled by the elastic properties of the solid. This leads to very different characteristics. For instance, the particle motion for Rayleigh waves is *retrograde* while for waves in water particle motion is *prograde*, which means that the particles orbit in opposite directions. Also unlike water waves the vertical component of particle motion in a solid is greater than the horizontal component. Both components decay exponentially with depth so that most of the energy associated with Rayleigh waves is confined near the surface to a depth roughly equal to the wavelength.<sup>33</sup> From a practical point of view the implication is that R-waves will not be attenuated by barriers (natural or man-made) that are small compared with the wavelength, but more on this later.

Pioneering work of Lamb<sup>34</sup> on the response of an isotropic elastic half space to different kinds of impulsive and harmonic loads forms the basis of all contemporary understanding of wave propagation in elastic half space. All three types of waves (P-, S-, and R-waves) are nondispersive. This means that their wave speeds are each independent of the excitation frequencies. In the ground, the P-wave speed is the highest, typically 400 to 800 m/s. The S-wave is somewhat slower than the P-wave and only slightly faster than the R-wave, typically 200 to 300 m/s. For the frequency range of interest, the distribution of energy between the three different kinds of waves are calculated by Miller and Pursey<sup>35</sup> for an elastic half space excited by a vertically oscillating rigid disk on the surface. Of the total input energy, 67% radiates as



**Figure 4** Wave propagation in an elastic half space from a vertical surface load showing R-, S-, and P-waves. (Reproduced from Ref 32, by permission.)

R-waves, 26% as S-waves, and 7% P-waves. As P- and S-waves spread with hemispherical wavefronts in the ground, their decay rate is inversely proportional to the distance from the source. R-waves on the surface spread on a circular wavefront and with a decay rate inversely proportional to the square root of distance from the source. This reduction in amplitude with distance is called *geometric decay*. Material damping will also influence the rate at which energy decays with distance from the track, and to estimate this *material decay* it is necessary to use a model for damping.

For surface roads and railways and for buildings with reasonably shallow foundations, it is generally thought that the Rayleigh wave is chiefly responsible for vibration propagation. But this thinking is misguided in almost all situations due to the effects of layered media, wave reflections from bedrock, and the effects of building foundations. Some of these effects have analytical solutions. The Stoneley wave, for instance, appears at the discontinuous interface within an infinite solid formed by bonding two different half spaces. The Love wave (the fastest of all surface waves) moves within a surface layer bonded to a half space. The motion of particles is in a horizontal plane parallel to the free surface. Love waves can cause vibration to travel long distances from a source of disturbance. Practically speaking, real ground conditions are typified by continuous variation of soil properties with depth, and geological layers are generally inclined or discontinuous. Even if the dynamic loads applied to the track are purely vertical, the transfer of energy at the various tunnel-soil and soil-soil interfaces will cause all wave types to be excited. The depth of the water table is a further complicating and seasonally varying factor. Analytical solutions are unlikely to be of much use to practicing engineers, but an understanding of the underlying physics is useful for vibration propagation over long distances from the source. It

is useful to know that surface waves are of significance because they carry most of the energy and their geometric decay rate is low.

For propagation over long distances it is necessary to include a realistic model for damping. Modeling of damping is a matter of personal preference, and the most common approach is to use a material loss factor.<sup>36</sup> But given that many affected by vibration from railways live very close to the railway, it is not often necessary to be concerned about material decay. More important, however, are details of track components that may reduce the proportion of energy that enters the ground. For instance, energy that propagates along the rails may be dissipated before being transmitting into the ground. In tunnels the propagation process is more complicated due to interaction between the track, tunnel, and surrounding soil. It can readily be seen that vibration prediction from railways is not straightforward, but practicing engineers have evolved a number of methods for dealing with the various complications.

## 5 PREDICTION METHODS

There is increasing public sensitivity to noise and vibration, and transportation systems need to conform to increasingly stringent legislation. As a result engineers need accurate methods for predicting noise and vibration from road and rail systems. This section focuses on two prediction methods, while a good review of other methods can be found in the work of Leventhall<sup>11</sup> and Remington.<sup>12</sup>

The first is a method developed by Nelson and Saurenman.<sup>37</sup> It aims to estimate ground-borne noise and vibration from trains for a variety of building types, soil conditions, and track designs in the frequency range of 6.3 to 200 Hz in third octave bands. Design features such as resilient direct-fixation fasteners, floating slab track, resiliently supported sleepers,

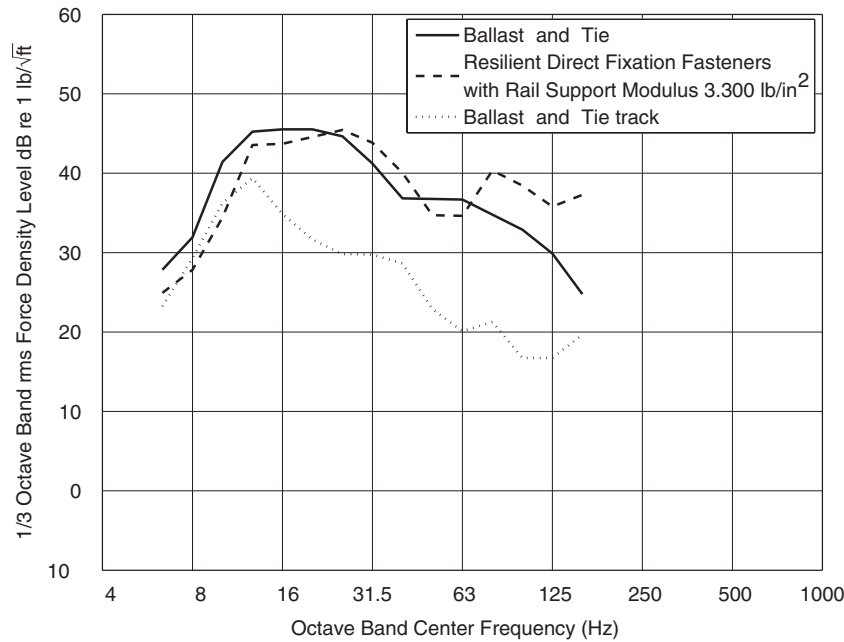


Figure 5 Force densities for heavy rail transit vehicles. (Reproduced from Ref. 37 with permission.)

continuous welded rails, vehicle suspensions, and others are considered. The method predicts the level of noise and vibration at a building (existing or proposed) at some distance from an existing or proposed surface railway or subway. The force density at trackbed, calculated from field measurements, are provided as a set of curves for different types of trains and track systems; see Fig. 5. The unit of force density is force divided by the square root of train length. The force density represents an incoherent line source at the trackbed. The curves may not correspond exactly to the actual train track system under consideration, in which case adjustments are provided for variations including floating slab, ballast mat, and primary suspension stiffness. The next step in the method is to conduct a field test procedure to determine the response due to a line source by measuring a point-to-point transfer mobility (Green's function) from the railway to the ground surface. The transfer mobility can be calculated by striking the ground or the subway invert with a hammer and measuring the input force and the resulting vibration on the ground.

For a proposed subway, the hammer is struck at the bottom of a borehole drilled to the depth of the proposed tunnel. The transfer mobility of a point source is numerically integrated over the train length to calculate the response due to a line source representing the train. This adjusted force density and the calculated response due to a line source are used to determine the vibration at the location of the building foundation as if the building were not present. A *coupling loss correction* is then used to calculate the vibration at the building foundation for the frequency range of

interest. The coupling loss gives the third octave band curve of the actual foundation vibration relative to the level of incident ground surface vertical vibration that would exist in the absence of the building and its foundation. The vibration at the foundation is corrected by means of *floor-to-floor attenuation* and *floor resonance amplification* to obtain the third octave band of vibration at the floor under consideration. These predictions can also be used to estimate room noise levels.

A second—and very different—method developed by Madshus et al.<sup>2</sup> predicts the vibration velocity from railway trains using the following formula:

$$V = F_V F_R F_B \quad (1)$$

where  $F_V = V_T F_S F_D$  is the basic vibration function.  $V_T$  is a train-type specific vibration level, defined as the vibration level on the ground at a reference distance of  $D_0 = 15$  m from the center of the track, when a train of the specified category passes at reference speed of  $S_0 = 70$  km/h on a standard track and embankment.  $F_S = (S/S_0)^A$  is a speed factor, which accounts for the effect of the train speed  $S$  and  $A$  is the train speed exponent.  $F_D = (D/D_0)^{-B}$  is a distance factor that accounts for the distance attenuation due to geometrical and hysteric damping.  $D$  is the distance from the center of the track to the receiver and  $B$  is the distance exponent.  $F_R$  is the track quality factor and  $F_B$  is the building amplification factor; it is used to transform the free-field ground vibration to floor vibration at the most unfavorable place.

**Table 2** Typical Parameters for Prediction Method

| Ground Conditions | Train Type | $V_T$ (mm/s) | A       | B       |
|-------------------|------------|--------------|---------|---------|
| Soft clay         | High speed | 0.4–0.5      | 0.9–1.1 | 0.3–0.8 |
|                   | Freight    | 0.7–0.8      | 0.9–1.1 | 0.3–0.4 |
| Medium clay       | High speed | 0.1–0.15     | 0.9–1.1 | 0.9–1.0 |
|                   | Freight    | 0.2–0.25     | 0.9–1.1 | 0.4–0.6 |

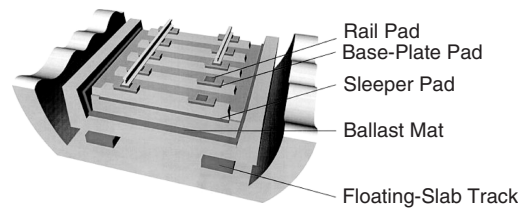
Source: From Ref. 2, reprinted by permission.

Equation (1) is used to predict one-third octave r.m.s values, with all factors being frequency dependent. Also a simplified version can be used at which all factors are frequency independent and directly related to the frequency-weighted root-mean-square (rms) velocity. The parameters of the model are computed based on a regression analysis. Typical parameters are given in Table 2.

There are other methods for predicting vibration from roads and railways. Examples are the prediction of loading and road response by Rudder<sup>38</sup> and the analytical-empirical ground vibration technique developed by Ungar and Bender.<sup>39</sup> These methods all have various merits. Individuals responsible for making vibration predictions will need to balance the need for accuracy against the resources (financial and computational) available. Some consultants will have access to an extensive database of vibration measurements from different sites, and their own experience will substantially enhance the usefulness of the predictive methods.

## 6 VIBRATION COUNTERMEASURES

Ballast in conventional railway track can act effectively to reduce vibration propagation. Generally, vibration countermeasures are required in confined spaces where ballast cannot be employed effectively. It is common to experience vibration problems where the rail is directly fixed to concrete or steel structures such as tunnels, bridges, and roadways. Vibration can also be controlled by careful attention to the detail of rail alignment, track foundation, widening of embankment (for surface railways), heavy tunnel structure, and increased tunnel depth.<sup>12,13,40</sup> Vibration can also be reduced by controlling rail and wheel roughness. This is generally considered a maintenance issue rather than a vibration countermeasure. One exception is that roughness growth can be controlled by the implementation of lubrication, especially at curves. In general, though, rail and wheel grinding are required to keep rail roughness under control. Moreover, all vibration countermeasures will require maintenance and inspection for their continued effectiveness, and some countermeasures require less maintenance than others. It must always be born in mind that any alteration to the track may render it more susceptible to corrugation growth, and this may more than outweigh the benefit of vibration countermeasures.<sup>25</sup> This can work both ways: Too high a rail support stiffness may not only give poor isolation but may contribute to excessive rail corrugation and thus excessive vibration and noise.<sup>37</sup>

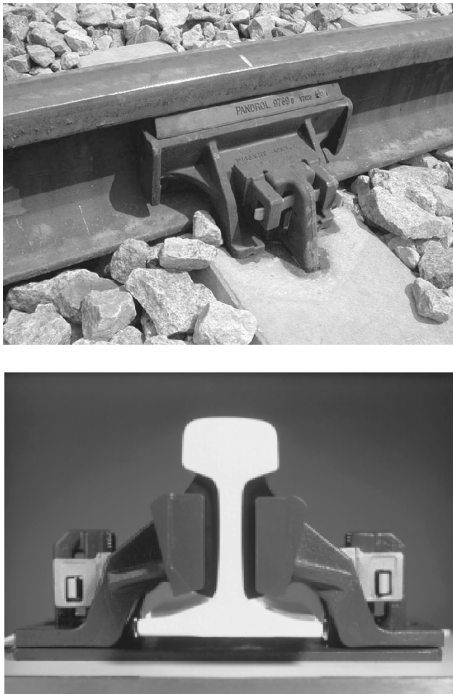


**Figure 6** Various rubber products used to reduce noise and vibration transmission from railways. (Courtesy of Getzner Werkstoffe GmbH, Bludenz, Austria.)

Vibration countermeasures fall into four categories. First, certain aspects of vehicle design will reduce dynamic force levels, for instance, by using low-stiffness vehicle suspension and a low unsprung mass (axle, axlebox, and wheel). Reducing the unsprung mass of a bogie can lead to a reduction of up to 10 to 15 dB in levels of ground-borne vibration.<sup>40</sup> Second, vibration can be isolated at the source by means of resiliently supported track (e.g., railpads and other resilient track support, resiliently mounted sleepers, ballast mat, embedded rail, and floating slab track) as illustrated in Fig. 6. Other in-track measures include reduced sleeper spacing and sleeper-mass redistribution.<sup>13</sup> Third, vibration propagation can be interrupted, for example, by digging a trench in the vibration path between the source and the receiver. This is an area of extensive study,<sup>32,41</sup> but two broad conclusions can be stated here. The performance of open trenches is better than that of concrete-filled trenches, and trench geometry has a clear effect on the isolation performance. Finally, vibration isolation can be achieved at the receiver by using rubber bearings or steel springs in the building foundations.

The most economic solution for vibration problems in old underground tunnels with directly fixed or ballasted track is likely to be resilient track support. The use of railpads does not raise the height of the rail significantly, which is an important consideration in small tunnels. A recent advance in resilient rail technology involves supporting the rail by the web and the underside of the head with large rubber wedges, leaving the foot of the rail suspended,<sup>20</sup> as shown in Fig. 7. This provides very low vertical dynamic stiffness while maintaining lateral stiffness required for vehicle and rail stability. The resilient bearing again has a low profile, and it can be installed and removed easily for maintenance. One possible disadvantage of using soft pads is that the level of noise radiated from the rail may be increased on account of the resiliently supported rail being less constrained by the base plates.<sup>13</sup>

Floating slab track is widely known as an effective measure of vibration isolation for underground tunnels. The track is mounted on a concrete slab that is fixed to the tunnel invert by means of rubber bearings or steel springs. The slab may be continuous or discrete (segmented). Continuous slab is cast in situ and segmented slab is constructed in discrete precast



**Figure 7** Resilient support of a rail, the Vanguard system. (Courtesy of Pandrol Ltd, UK.)

sections. Discrete slab is convenient in many situations: It has the advantage that vibration propagation along the track is interrupted, but there is potential for maintenance problems if the slab segments are to move relative to each other, affecting track alignment. Continuous slab has the advantage of structural integrity, but it permits vibration to propagate along the tunnel, and this may lead to greater levels of radiation from the tunnel. Examples of floating slab tracks are the 1.5-m slab in Toronto, the 3.4-m Eisenmann track in Munich and Frankfurt, the 7-m slab in the New York City subway, and the British VIPACT continuous-slab system.

In circumstances where ballast is used, as is generally the case for surface trains and in some tunnels, the use of ballast mats can be very effective. A ballast mat is simply a mat of resilient material (usually rubber, but sometimes rockwool inserted underneath the ballast). It is analogous to the use of floating-slab track in underground tunnels. An example of the use of ballast mats within a tunnel can be found in the MBTA Boston in the United States. Ballast mats are also used to improve electrical isolation, water drainage, or to reduce pulverization of the ballast.<sup>12</sup> The mechanism by which ballast mats attenuate vibration is not well understood. One view is that the ballast mat acts as a spring supporting the mass of ballast leading to vibration isolation.<sup>42</sup> An alternative view is that of the authors who believe that ballast mats are effective

on account of their ability to allow ballast to move under the application of axle loading. This prevents the ballast from “locking up.” The truth is, no doubt, a combination of these two effects.

In soft ground it may be necessary to improve the characteristics of the soil surrounding the track. Soil stabilization by means of injection grouting under and adjacent to the track is effective because it stiffens the soil structure, which reduces vibration propagation due the quasi-static effect.<sup>13</sup> This can be particularly important for high-speed trains where trans-Rayleigh effects can cause large-amplitude vibration.

A common theme that runs through all attempts to assess the performance of vibration countermeasures is that trends are difficult to identify. It is found, for instance, that measures effective for one site are less effective for another and that the effectiveness of in-track countermeasures is sometimes found to reduce with distance from the track. It is clear that detailed modeling is required to make sense of these observations and that accurate models can be used to drive the development of new countermeasures.

## 7 MODELING

The strategy for modeling ground vibration from road traffic differs from that for rail traffic mainly because of the difference in excitation mechanisms. For road vehicles the pavement can be considered to be rigid, and the vehicle suspension is responsible for controlling the dynamic forces generated in response to road roughness. This permits a decoupling of the excitation mechanism from the wave propagation mechanism, which simplifies modeling. However, for rail vehicles the unsprung mass of wheel and axle combine with the mass of that part of the rail within the deflection bowl to move in response to rail roughness. Decoupling is no longer possible. Nevertheless, the propagation mechanisms are largely common to both problems, and an understanding of wave propagation and modeling of the response of a half space under different kinds of loading is essential before embarking on any detailed modeling. Much of the fundamental work was discussed in Section 4, but there are some further fundamental issues that require discussion. These include: (1) the effects of moving loads, (2) random vibration theory, (3) track modeling, (4) tunnel modeling, and (5) ground modeling. For any realistic model of traffic-induced vibration, all these effects will need to be considered. Most can only be addressed through extensive use of mathematical techniques (Fourier, wavelet, and Floquet transforms) and through the use of computational techniques such as the finite element and boundary element methods (FEM–BEM). It is not possible here to do more than to identify the various techniques.

Early models for computing vibration generated from moving loads used a solution of moving constant force on elastic half space. More advanced developments were made possible by including the inertial effect of vehicles. Fryba<sup>43</sup> discusses lucidly several aspects of vibration of solids and structures under moving loads. Moving loads can be classified in three

ways depending on whether the speed is less than, equal to, or greater than the lowest wave speed in the structure—that is, the critical speed. Above the critical speed, waves analogous to shock waves are generated, producing large amplitudes in the structure. These large amplitudes are only to be expected in soft ground or on flexible structures; most readers need not concern themselves with the critical speed.

A theoretical analysis of the transmission of road-vehicle-induced vibration through the ground by Hunt<sup>44</sup> considers vibration as a Gaussian-distributed random process. Vehicles are modeled as single-axle two degrees of freedom. Power spectral expressions are calculated for the vertical ground acceleration away from a long straight road. Only a half-space model is used (i.e., no layering), and approximate methods are used to account for the effect of the bending stiffness of the road surface. A series of measurements is presented to validate the model, and a parametric survey demonstrates the sensitivity of traffic-induced ground vibration levels to ground properties, vehicle speed, traffic intensity, road roughness, and vehicle characteristics. This methodology is extended by Hunt<sup>45</sup> for the calculation of building vibration generated by railways using theory of random vibration and models of infinite length for both building and track.

The Winkler beam<sup>46</sup> (or *beam on elastic foundation*) is used extensively to model railway tracks. Discrete supports of rails can be assumed as an elastic foundation of Winkler type. The benefit of using Winkler beam theory to model railway track is that it is relatively easily coupled to models of the ground. But while Winkler beam methods are within the computational abilities of many engineers, they do not enable the beneficial effects of track resilience to be computed with any accuracy. It is not unusual for insertion loss performance figures of 30 dB or better to be quoted based on Winkler theory, but it is generally found that such performance cannot be achieved in practice. At any reasonable distance from the track, it is unrealistic to expect insertion loss of greater than 15 dB from even the best of vibration countermeasures.

A theoretical model for both the generation and propagation of vibration from freight trains is presented by Jones and Block.<sup>1</sup> The model accounts for both the rail roughness and the quasi-static effect of moving axles. The input to the model is taken from measurements by using a track recording coach. The predicted vibration agrees well with the measured vibration in the frequency range of 5 to 30 Hz. It is found that heavy freight trains emit ground vibration with predominant frequency component in the range of 4 to 30 Hz.

Sheng et al.<sup>23,47–49</sup> present a fully three-dimensional model of a railway track coupled to a multilayered half space. Both of the rails are modeled as a single Euler–Bernoulli beam, and railpads are modeled as a continuous resilient layer. Sleepers are modeled as a continuous beam with no bending stiffness, and ballast is modeled as a continuous layer of a linear spring with a consistent mass approximation. The results show a good agreement with measurements taken at three different

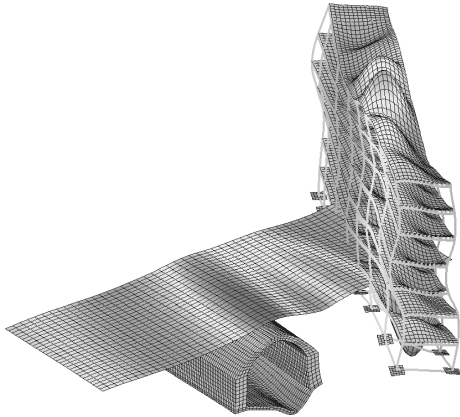
sites and demonstrate the dominance of the quasi-static effect near the track at low frequencies. The model is used to investigate the effect of track properties on ground-borne vibration.

Krylov<sup>50</sup> presents a model to investigate the effect of high-speed trains on ground vibration. The important feature of the model is that it accounts for sleeper spacing and the quasi-static forces of the moving train. This model is used by Krylov et al.<sup>29</sup> to predict the vibration generated by the French TGV and Eurostar high-speed trains along tracks built on soft soils. Degrande and Lombaert<sup>51</sup> use the Betti–Rayleigh dynamic reciprocity theorem to increase the efficiency of Krylov's model. Degrande and Schillemans<sup>51</sup> compare the results from this model with free-field vibration measurements during the passage of a Thalys high-speed train at various speeds. The model gives good predictions at low frequencies where the quasi-static response dominates and at high frequencies where sleeper-passing response dominates. At the mid-frequency range where other mechanisms, for example, rail roughness are important, the model underestimates the response. Another theoretical and experimental study on vibration from high-speed trains is presented by Kaynia et al.<sup>28</sup> where a good match is shown between the predicted and measured results.

A three-dimensional semianalytical model for a floating-slab track in a deep underground tunnel is presented by Hunt.<sup>53</sup> The model consists of Euler–Bernoulli beams to account for the rails and the track slab. The slab is coupled to the *pipe-in-pipe model*, which consists of a thin shell (the inner pipe) representing a tunnel, embedded within an infinite continuum with a cylindrical cavity (the outer pipe) representing the surrounding soil. Coupling is performed in the wavenumber–frequency domain. Three-dimensional numerical models are developed by some researchers to model vibration from underground railways. The main disadvantage of these models is that they are computationally expensive. Powerful numerical models are developed under CONVURT, where a coupled FEM–BEM is used. The FEM is used to model tunnel walls while the BEM is used to model the surrounding soil. Taking account of periodicity in the tunnel direction using Floquet transformation makes a significant improvement in the running time.<sup>54</sup>

Coupled FEM/BEM are also used to model road-traffic-induced vibration. An example is the work of Pyl et al.,<sup>55</sup> which incorporates a moving source and accounts for soil–structure interaction. A two-dimensional vehicle model is used on longitudinal road roughness to account for the source, and the input is calculated by assuming the vehicle response to be uncoupled from the road–soil response. The finite element method is used to model a building where the vibration is predicted, and the boundary element method is used to model the ground. An example of the power of coupled FEM/BEM methods is illustrated in Fig. 8.

A different numerical method is used by Sheng et al.<sup>56</sup> This method is the *discrete wavenumber*



**Figure 8** Example of boundary element modeling. Vibration of a building near a Metro tunnel in Paris. (Courtesy of D. Clouteau, École Centrale, Paris.)

*fictitious force* method and is used to model an underground tunnel embedded in a half space. The method depends on writing the boundary integral equations of only the displacement Green's function. This is an advantage over the BEM as the traction Green's function is not required. The finite difference method<sup>57</sup> may also be used to model the traffic-induced and rail-induced ground vibration. Its advantage is thought to be that the computational code is much simpler than conventional numerical methods such as BEM and FEM.

The methods described here are of limited use in practice for four reasons: First, they require substantial computational resources in addition to a working knowledge of some fairly high powered analytical tools. Second, a large quantity of input data is required for the models, and this may not be readily available. In particular, the variation of soil properties with depth is unlikely to be known to great accuracy, and the quality of predictions made can only be as good as the quality of the input data. Third, few of these models are validated over a wide range of different track and ground conditions. It may be more sensible to use an older and simpler predictive method that has stood the test of time. Finally, no models presently available take into account the nonlinearities of track elements and of soils. These are likely to be significant if accurate predictions are to be made.

## 8 CONCLUSION

There is no simple answer for the computation of vibration generated by road or rail traffic. Simple models do not work with sufficient accuracy and elaborate models are computationally expensive. Empirical models are valid only in certain simplified circumstances and experimental measurement of vibration is only really useful when there is the option of trying out various different methods. It is the view of the authors that substantial advances in vibration reduction will only occur when the computational tools are widely

available and in the hands of the engineers responsible for road, track, and tunnel design.

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